

A macroscopic challenge for quantum spacetime

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Over the last decade a growing number of quantum-gravity researchers has been looking for opportunities for the first ever experimental evidence of a Planck-length quantum property of spacetime. These studies are usually based on the analysis of some candidate indirect implications of spacetime quantization, such as a possible curvature of momentum space. Some recent proposals have raised hope that we might also gain direct experimental access to quantum properties of spacetime, by finding evidence of limitations to the measurability of the center-of-mass coordinates of some macroscopic bodies. However I here observe that the arguments that originally lead to speculating about spacetime quantization do not apply to the localization of the center of mass of a macroscopic body. And I also analyze some popular formalizations of the notion of quantum spacetime, finding that when the quantization of spacetime is Planckian for the constituent particles then for the composite macroscopic body the quantization of spacetime is much weaker than Planckian. These results show that finding evidence of spacetime quantization with studies of macroscopic bodies is extremely unlikely. And they also raise some conceptual challenges for theories of mechanics in quantum spacetime, in which for example free protons and free atoms should feel the effects of spacetime quantization differently.

I. INTRODUCTION AND MOTIVATION

Traditionally the quantum-gravity problem was studied as a mere technical exercise, assuming that it might be impossible to find experimental evidence of the minute effects produced by the characteristic length scale of quantum gravity, expected to be of the order of the Planck length $\ell_P \simeq 10^{-35}m$. This changed over the last decade as a result of a growing number of studies (see, *e.g.*, Refs. [1–11]) showing that evidence of Planck-length quantum properties of spacetime might be within our experimental reach if we exploit some candidate indirect manifestations of spacetime quantization. An intuitive example of candidate indirect manifestations of spacetime quantization is found in results showing that certain ways to introduce the Planck length as scale of spacetime quantization admit a dual picture in which the Planck length also plays the role of scale of curvature of momentum space, with implications for relativistic kinematics (see, *e.g.*, Refs. [11, 12]).

It would of course be important to also find opportunities for observing Planck-length spacetime quantization directly. And according to the studies recently reported in Refs. [13, 14] this might be possible, at least in the sense that we can achieve Planckian accuracy in measurements pertaining the center-of-mass coordinates of some macroscopic bodies¹. The study reported by Pikovski *et al* in Ref. [13] focuses on the center-of-mass motion of a mechanical oscillator, while the study reported by Bekenstein in Ref. [14] focuses on the center-of-mass motion of

a macroscopic dielectric block traversed by a single optical photon.

In attempting to assess the likelihood of success of these proposals I noticed that they involve small momentum transfer from a low-energy photon to a macroscopic body, the body being describable fully within the “nonrelativistic limit” (small velocities, where Galilean relativity holds). And I find that the arguments that inspired quantum-gravity research on Planck-length spacetime quantization do not apply to such interactions between soft photons and macroscopic bodies. The current consensus among theorists (see, *e.g.*, the reviews in Refs. [16, 17]) is that Planck-length spacetime quantization is needed because any attempt to localize a particle with Planckian accuracy requires concentrating energy of order the inverse of the Planck length within a Planck-length-size region, and in such situations our present understanding of gravitational phenomena suggests that a black hole should form, rendering the localization procedure meaningless. The procedures proposed in Refs. [13, 14] for Planck-length accuracy in the control of the center-of-mass position of a macroscopic body evidently do not involve any particularly high concentration of energy in small regions, certainly no inverse-Planck-length energy in any Planck-length-size region.

The hope that the center of mass of a macroscopic body might be subject to the same Planck-length quantum properties of spacetime expected for fundamental particles is therefore evidently based on an implicit inductive argument: the necessity of Planck-length spacetime quantization arises exclusively in arguments involving fundamental particles, but once that is accommodated in the theory perhaps by some (unproven and unknown) consistency criterion the Planck-length quantum properties of spacetime would also affect a macroscopic body. To my knowledge this huge extrapolation is not confirmed by any known results of quantum-spacetime

¹ The issues I here raise turn out to be relevant also for studies such as the one Mercati, Laemmerzahl, Tino and myself reported in Ref. [15], probing center-of-mass-motion properties of Cs and Rb atoms: according to my line of analysis a large atom could qualify as macroscopic in quantum-spacetime research.

research. On the contrary I here provide a simple argument suggesting that this extrapolation is incorrect. I consider a few of the models being studied in the quantum-spacetime literature, with my selection criteria for models being the availability of a characterization in terms of deformed commutators (which allows my analysis to advance very straightforwardly). And my way to probe conceptually the issue here at stake is centered on a simplified characterization of the center of mass of a body composed of N constituent particles. I take as center-of-mass coordinates the observables X, Y, Z , with

$$X = \frac{1}{N} \sum_{n=1}^N x_n, \quad Y = \frac{1}{N} \sum_{n=1}^N y_n, \quad Z = \frac{1}{N} \sum_{n=1}^N z_n \quad (1)$$

(where of course x_n, y_n, z_n are the coordinates of the n -th composing particle), and I take as center-of-mass momentum the observables P_x, P_y, P_z , with

$$P_x = \sum_{n=1}^N p_{x,n}, \quad P_y = \sum_{n=1}^N p_{y,n}, \quad P_z = \sum_{n=1}^N p_{z,n} \quad (2)$$

(where of course $p_{x,n}, p_{y,n}, p_{z,n}$ are the momentum components of the n -th composing particle).

This simplified description of a macroscopic body is sufficient for my purposes since the relevant phenomenological opportunities are for macroscopic bodies in the nonrelativistic regime and my main objective is to provide a counter-example to the conjecture that Planck-length quantum properties of spacetime apply in undifferentiated way to fundamental particles and macroscopic bodies composed of many particles. I shall show that the conjecture is false by showing very explicitly that it does not apply to macroscopic bodies whose center-of-mass motion is characterized by (1)-(2). And (1)-(2) is appropriate for macroscopic bodies whose constituents all have the same mass and whose center-of-mass degrees of freedom decouple from the other degrees of freedom.

II. RESULTS FOR CLASSICAL SPACETIME AND LIE-ALGEBRA QUANTUM SPACETIME

Let me first remind my readers of the mechanism through which the description (1)-(2) gives satisfactory results within ordinary quantum mechanics, in classical spacetime, where the only non-trivial commutator is Heisenberg's

$$[x, p_x] = i\hbar$$

where I focused for simplicity on the x -direction.

Evidently the Heisenberg commutator also applies to a body's center of mass described by (1)-(2):

$$\begin{aligned} [X, P_x] &= \left[\frac{1}{N} \sum_{n=1}^N x_n, \sum_{m=1}^N p_{x,m} \right] \\ &= \frac{1}{N} \sum_{n=1}^N \sum_{m=1}^N \delta_{n,m} i\hbar = \frac{1}{N} \sum_{n=1}^N i\hbar = i\hbar \end{aligned} \quad (3)$$

This is a trivial (or at least well known) success of the description I am using as conceptual probe, since it applies to the familiar context of ordinary quantum mechanics in classical spacetime.

My next application is already non-trivial and novel, but nonetheless provides further elements in support of the usefulness of the conceptual probe I am using, centered on (1)-(2). For this I consider a class of quantum-spacetime pictures involving noncommutativity of coordinates of Lie-algebra type [18–20]

$$[r^\alpha, r^\beta] = i\ell\theta_\gamma^{\alpha\beta} r^\gamma$$

with² $r^1 = x, r^2 = y, r^3 = z$.

This is just one of the types of noncommutativity of coordinates that are being considered. It is here particularly significant since it is the only case where the literature does provide a suggestion that macroscopic bodies might be affected by Planck-length features differently from their constituent particles. These Lie-algebra spacetimes are known to be dual to momentum spaces with curved geometry [11, 12] and one of the implications is that the laws of conservation of momentum for fundamental particles are Planck-length deformed. It was noticed in Ref. [22] that applying the relevant deformed conservation laws to the constituents of a macroscopic body would give a net result for collisions among macroscopic bodies (when described as the combined result of a large number of collisions among constituents) such that momentum conservation for macroscopic-body total momentum would be affected by weaker corrections than momentum conservation for the particle constituents: Ref. [22] observed that such momentum-conservation analyses suggest that the curvature of momentum space felt by the macroscopic body is not the Planck length but rather the Planck length divided by the number N of particle constituents.

So for Lie-algebra spacetimes the literature provides at least a suggestion, based on the dual momentum-space picture, that the effective Planck length should be rescaled by N for macroscopic bodies. Remarkably my simple “conceptual probe” produces for the noncommutativity of coordinates exactly the same scaling with N of the spacetime-quantization length scale. To see this in the simplest possible way let me consider the case of a commutator of type

$$[x, y] = i\ell r^\alpha$$

with α taking any value among 1, 2, 3 (so that essentially I consider at once cases of the type $[x, y] = i\ell x$ and of the type $[x, y] = i\ell z$).

² I focus on spatial noncommutativity, which is sufficiently general for establishing the issue for macroscopic bodies which is here of interest. One may also consider (see, *e.g.*, Ref. [18]) the more exotic case of spacetime noncommutativity with also the time coordinate as noncommutative, but that added complication (which would impose working within the covariant formulation of quantum mechanics [21]) is unnecessary for my purposes.

Applying $[x, y] = i\ell r^\alpha$ to the constituent particles of a macroscopic body one then finds for center-of-mass coordinates of type (1) the result

$$\begin{aligned} [X, Y] &= \left[\frac{1}{N} \sum_{n=1}^N x_n, \frac{1}{N} \sum_{m=1}^N y_m \right] \\ &= \frac{1}{N^2} \sum_{n=1}^N \sum_{m=1}^N \delta_{n,m} i\ell r_n^\alpha = \frac{1}{N^2} \sum_{n=1}^N i\ell r_n^\alpha = i \frac{\ell}{N} R^\alpha \end{aligned} \quad (4)$$

where of course $R^\alpha \equiv N^{-1} \sum_{n=1}^N r_n^\alpha$.

Evidently (5) shows that the effects of Lie-algebra coordinate noncommutativity for macroscopic bodies are scaled down by a factor of $1/N$, as already suggested by the dual picture on the associated momentum space given in Ref. [22].

Most importantly for the scopes of this Letter my Eq. (5) provides a first piece of evidence of the fact that spacetime quantization should be felt differently by macroscopic bodies with respect to their constituents, with the center of mass of macroscopic bodies being affected by weaker spacetime-quantization effects.

III. RESULTS FOR OTHER QUANTUM-SPACETIME PICTURES

I shall now show that my perspective on macroscopic bodies in a quantum spacetime has applicability that goes beyond the specific context of Lie-algebra spacetime noncommutativity. My next example is the one of “Moyal noncommutativity”, with coordinate-independent commutators of the coordinates, such as

$$[x, y] = i\ell_M^2 \quad (5)$$

This is perhaps the most studied candidate scenario for the quantization of spacetime (see, *e.g.*, Refs. [23, 24] and references therein). To my knowledge there is no result in the literature anticipating that macroscopic bodies should perceive this noncommutativity differently from their constituents, but this is what I find applying simply (1) for the center-of-mass coordinates with the constituents governed by noncommutativity (5):

$$\begin{aligned} [X, Y] &= \left[\frac{1}{N} \sum_{n=1}^N x_n, \frac{1}{N} \sum_{m=1}^N y_m \right] \\ &= \frac{1}{N^2} \sum_{n=1}^N \sum_{m=1}^N \delta_{n,m} i\ell_M^2 = \frac{1}{N^2} \sum_{n=1}^N i\ell_M^2 = i \left(\frac{\ell_M}{\sqrt{N}} \right)^2 \end{aligned} \quad (6)$$

Therefore also for the Moyal case the noncommutativity of center-of-mass coordinates should be weaker than the noncommutativity of the coordinates of the constituents. Specifically the Moyal noncommutativity length scale ℓ_M gets reduced by a factor of $1/\sqrt{N}$.

Another much studied class of quantum-spacetime pictures that I should consider is the one that does not

invoke noncommutativity of coordinates, but is instead centered on modifications of the Heisenberg commutator of the general type [25, 26]

$$[x, p] = i\hbar(1 - \lambda' p + \lambda^2 p^2) \quad (7)$$

Even with commuting coordinates these modifications of the Heisenberg commutator produce spacetime quantization. The key role for this is played by the parameter λ^2 of the quadratic term. The standard Heisenberg commutator still allows localizing a particle sharply at a point ($\delta x \rightarrow 0$) if $\delta p \rightarrow \infty$, *i.e.* if all information on the conjugate momentum is given up. But for $\lambda^2 \neq 0$ the Eq. (7) produces a see-saw formula [25, 26] such that δx receives a novel contribution proportional δp in addition to the standard Heisenberg term going like $1/\delta p$, in such a way that the coordinate x cannot ever be measured sharply, as required for a quantum-spacetime picture.

Of some interest for my thesis is also the perspective given in Ref. [26], advocating the specific choice of $\lambda' = \lambda$ in (7), partly because of its consistency (in the sense of Jacobi identities) with commutativity of coordinates among themselves and of momenta among themselves.

Keeping these facts in mind it is then interesting to look at the properties of a center of mass described by (1)-(2) when the constituents are governed by (7):

$$\begin{aligned} [X, P_x] &= \left[\frac{1}{N} \sum_{n=1}^N x_n, \sum_{m=1}^N p_{x,m} \right] \\ &= \frac{1}{N} \sum_{n=1}^N \sum_{m=1}^N \delta_{n,m} i\hbar(1 - \lambda' p_{x,m} + \lambda^2 p_{x,m}^2) \\ &= i\hbar \left[1 - \frac{\lambda'}{N} P_x + \frac{\lambda^2}{N^2} P_x^2 + \frac{\lambda^2}{N^2} \sum_{n=1}^N (P_x^2 - N^2 p_{x,n}^2) \right] \\ &\simeq i\hbar \left(1 - \frac{\lambda'}{N} P_x + \frac{\lambda^2}{N^2} P_x^2 \right) \end{aligned} \quad (8)$$

where for the last approximate equality I restricted my attention to macroscopic bodies in (quasi-)rigid motion, as those of interest for the mentioned experimental proposals put forward in Refs. [13, 14], so that one can expect for every n that $p_{x,n} \simeq P_x/N$ (up to small and uncorrelated deviations).

Evidently also for quantum spacetimes characterizable in terms of Eq.(7) I am finding that the center of mass of a macroscopic body should be affected more weakly than its constituents by spacetime quantization. Notably my argument suggests that the length scales in Eq.(7), both λ' and λ , get scaled down by $1/N$. This appears to ensure in particular that the prescription $\lambda' = \lambda$ advocated in Ref. [26] could apply both to fundamental particles and to the center of mass of a macroscopic body (but in the macroscopic case both λ' and λ are reduced by $1/N$).

IV. IMPLICATIONS AND OUTLOOK

Because of the nature and simplicity of my conceptual probe centered on (1)-(2), it may still be legitimate to ask whether some special macroscopic bodies could be affected by spacetime quantization just as much as their constituents. But the evidence I here provided clearly shows that this cannot be generic. On the contrary, it should be hard to find even a single type of macroscopic body such that the deviations from (1)-(2) would conspire to compensate exactly the strong $1/N$ (or $1/\sqrt{N}$) suppression of spacetime quantization length scales which I here exposed.

And it does not take a particularly macroscopic system for my concerns to be applicable. Think of just bound systems of two identical particles, with coordinate vectors \vec{r}_1 and \vec{r}_2 and with bounding potential $V(|\vec{r}_1 - \vec{r}_2|)$ affecting only the relative motion: for such systems (1) and (2) are correct, with $N = 2$.

Evidently my line of analysis applies to a variety of scenarios for spacetime quantization which involve non-commutativity of spacetime coordinates and/or modifications of the Heisenberg commutators, either fundamentally or at some effective-theory level of description. It remains to be seen whether in other formalizations of quantum spacetime there is the same type of relationship between properties of center-of-mass coordinates and properties of the coordinates of the constituents of a macroscopic body. The Loop Quantum Gravity approach [27] provides a very popular path toward the formalization of quantum spacetime. Some results (see, *e.g.*, Refs. [28, 29]) appear to suggest that spacetime coordinates could be effectively noncommutative in a regime where Loop Quantum Gravity describes experimental setups such as those of Refs. [13, 14]. In light of my findings these results on emerging spacetime noncommutativity might play a pivotal role in shaping the priorities of a phenomenology based on Loop Quantum Gravity.

On the quantitative side one should notice that I here analyzed four candidate spacetime-quantization length scales, ℓ , ℓ_M , λ , λ' , finding that for three of them the compositeness suppression should go at least roughly like $1/N$, while only for the scale of the Moyal case, ℓ_M , the suppression has $1/\sqrt{N}$ behaviour. A simple dependence on N should only be expected for some forms of spacetime quantization and for bodies composed of identical particles, but still there might be some usefulness in categorizing quantum spacetimes in part according to this $N^{-\sigma}$ criterion.

My observations represent a challenge for the quantum-spacetime idea on the experimental side, since they show that success is very unlikely for experiments exploiting our ability to control the center of mass of a macroscopic body, such as those proposed in Refs. [13, 14]. I do believe that such experiments still need to be made, since no theory result can preempt experimental investigations. But at times when particularly tough choices of prioritization are imposed by science's budgets, my observations could be relevant also on the experimental side.

Perhaps even more severe are the technical challenges that, according to my analysis, the description of macroscopic bodies imposes on theory work on the quantum-spacetime idea. A first challenge comes from the fact that my analysis clearly shows that macroscopic bodies have quantum-spacetime properties different from those of their constituents, but it gives no indication of which constituents are those “fundamental enough” to be affected by the full strength of Planck-scale effects. Think for example of molecules: I am finding that molecules are affected more weakly by quantum-spacetime effects than the atoms within them, but Planck-length magnitude of quantum-spacetime effects should be assumed for atoms or for protons and neutrons within the nuclei of atoms? or for quarks?

And a second challenge would need to be faced even assuming this first challenge is eventually addressed in a given quantum-spacetime picture, so that actually the picture predicts the magnitude of quantum-spacetime effects for, say, protons and also predicts how much weaker than for protons the effects are for, say, Cs atoms. We would clearly need a completely new type of theory of mechanics, in which the spacetime properties of different particles are different. We should renounce to one of their key aspects of simplicity that survived previous evolutions of our formulation of the laws of physics: the general-relativistic description of spacetime, just like the special-relativistic one and the Newtonian one, is indeed such that the implications of spacetime for particle properties are independent of compositeness, and are therefore the same for protons and large atoms.

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